Theory II Exercise Sheet 13 M. Lindemann

1	2	3	4	Σ
4	4	3	2	13

1: PCP

(a) $Y = \{(aba, a), (ba, babab)\}$

$u_1 = aba$ $v_1 = a$

$u_2 = ba$ $v_2 = babab$

One has two possible starts for a derivation:
The first one starting with a's and the second one starting with b's.

$u_1 = a\underset{\uparrow}{b}a \rightarrow v_1 = a$

next is b
$\rightarrow v_2$

$u_1 u_2 = ababa$

$v_1 v_2 = ababab$

$\uparrow \rightarrow u_2$ required

$u_1 u_2 u_2 = \underbrace{aba}\underbrace{ba}\underbrace{ba}$

$v_1 v_2 v_2 = \underbrace{a}\underbrace{babab}\underbrace{babab}$ ↯

$\Longrightarrow$ starting with a cannot be solved!

Second try, starting with b:

$u_2 = ba$ $\rightarrow v_2 = babab$

$\hookrightarrow u_2$ required!

$u_2 u_2 = baba$ $\hookrightarrow v_2 = bababbabab$

↑ impossible do derive in/with us!

$\Longrightarrow Y_{(a)}$ has no solution!

✓

1 (b) Starting with 3 in

$u_1 = bab$ $\qquad$ $v_1 = ba$

$u_2 = aaabb$ $\qquad$ $v_2 = a$

$u_3 = ab$ $\qquad$ $v_3 = abbab$

from $Y_{(b)}$.

$u_3 = ab$ $\qquad$ $v_3 = abbab$ ↳ u_1 req!

→ $u_3 u_1 = abbab$ $\qquad$ $v_3 v_1 = abbabba$ ↳ u_1 req.!

$u_3 u_1 u_1 = abbabbab$ $\qquad$ $v_3 v_1 v_1 = abbabbaba$ ↳ choosing u_2

$$\begin{cases} u_3 u_1 u_1 u_2 = abbabbabaaabb \\ v_3 v_1 v_1 v_2 = abbabbabaa \end{cases}$$

↳ choosing v_3

$$\begin{cases} u_3 u_1 u_1 u_2 u_3 = abbabbabaaabbab \\ v_3 v_1 v_1 v_2 v_3 = abbabbabaaabbab \end{cases} = \text{equal}$$

$\Rightarrow$ $Y_{(b)}$ has at least one solution!

$\Rightarrow$ '3 1 1 2 3' ✓

2 : PCP and Decidability

(a) Remark from page 114:

"For one-element alphabet X, the PCP is decidable over X"

With respect to theorem 4.1 (th. of Rice) as the set S is not $\emptyset$ and not $\mathcal{F}_{B,B}$ (i.e. $\emptyset \subset S \subset \mathcal{F}_{B,B}$) one can conclude, that M_1 is undecidable! ✓

b) We know (theorem 5.7) that the PCP over X is undecidable for $|X| \geq 2$.

This means, there is no TM $\mathcal{T}$ that can solve PCP over X $|X \geq 2$ (i.e. $X = \{0,1\}$)

$\Rightarrow M_2 = \emptyset$ which is decidable! ✓

3 : Theorem of Rice for languages

S is a non-trivial set of formal languages.

We can encode set Γ^* to a binary strings $\in B^*$

hence we can Reduce Ho to $BW'(S) = \{ b w_{\mathcal{T}} \mid L(\mathcal{T}) \in S \}$.

Therefore we consider an arbitrary function Language!

$g \in \mathcal{F}_{\Gamma\Gamma}$ and an arbitrary TM $\mathcal{T}$ s.t.

$\mathcal{T}' = \mathcal{T}'(\mathcal{T}, g)$.

When applied to string $s' \in B^*$, $\mathcal{T}'(\mathcal{T}, g)$ operates as follows:

1. s' is ignored and $\mathcal{T}'$ operates as $\mathcal{T}$ applied to the blank tape
2. If $\mathcal{T}$ halts, then $\mathcal{T}'$ operates as $\mathcal{T}_g$ applied to s'

well ... -2

$\Rightarrow$ Ho $\Rightarrow$ undecidable

4 : Theorem of Rice

The problem is equivalent to exercise 3,
where $L(J)$ is basically range of values
of h_J-function of TM_J. According to
the theorem 4.1 (RICE) $BW(S)$, with
S ~~=~~ set of reg. languages, is undecidable