FFT $\omega_n = e^{2\pi i/n}$ $p(x) = ax+b$, $q(x) = cx+d$

$FFT([b,a,0,0],4)$: $p=[b,a,0,0]$; $p^0=[b,0]$; $p^1=[a,0]$; $FFT([b,0],2) = (FFT([b],1)+FFT([0],1), b-0)$

$FFT([a,0],2) = (a+0, a-0)$; $\omega=1: d_0 = b+a$; $\omega=i: d_1 = b+a\cdot i$; $\omega=-1: d_2 = b-a$; $\omega=-i: d_3 = b-ai$

$FFT(p,4) = (d_0, d_1, d_2, d_3)$; $FFT(q,4) = (d+c, d+ci, d-c, d-ci)$; $FFT(p\cdot q, 4) = ((b+a)(d+c), (b+ai)(d+ci), (b-a)(d-c), (b-ai)(d-ci))$

Interpolation: $FFT([\langle FFT(p\cdot q,4)\rangle],4) = (d_0, d_1, d_2, d_3)$; $a_0 = \frac{1}{4}\cdot d_0$; $a_1 = \frac{1}{4}\cdot d_3$; $a_2 = \frac{1}{4}\cdot d_2$

$a_3 = \frac{1}{4}\cdot d_2 \Rightarrow p\cdot q = a_3x^3 + a_2x^2 + a_1x + a_0$ $\boxed{O(n\log n) + O(n) + O(n\log n) = O(n\log n)}$

RSA p, q, e | $n = p\cdot q$ | $\varphi(n) = (p-1)(q-1)$ | $e\cdot d + k\cdot\varphi(n) = 1 = ggT(e, \varphi(n)) \Rightarrow$ (gcd)

$\varphi(n) = x_1\cdot e + r_1$ $\quad r_2 = e - x_2\cdot r_1$

$e = x_2\cdot r_1 + r_2 \Rightarrow 1 = e - x_2\cdot(\varphi(n) - x_1\cdot e) = (x_1\cdot x_2 + 1)e - x_2\cdot\varphi(n)$ (with $d = x_1\cdot x_2+1$, $k = -x_2$)

$r_1 = x_3\cdot r_2 + 0$

$P = (e,n)$, $S = (d,n)$, P = public key | $P(M) = M^e \bmod n$, $S(C) = C^d \bmod n$

Universal Hashing | class H: $x \neq y$: $\frac{|\{h\in H : h(x) = h(y)\}|}{|H|} \leq \frac{1}{m}$ | $h: U \to [0..m-1]$, $U = [0..N-1]$

$H = \{h_{a,b}(x) \mid 1 \leq a < N \wedge 0 \leq b < N\}$

N prime | $h_{a,b}(x) = ((ax+b) \bmod N) \bmod m$ | collisions $\exists c\in\mathbb{N}: a(x-y) = cN \wedge (x-y) \mid N \Leftrightarrow (x-y) = \ell\cdot k$

$O(1+\frac{n}{m})$ | $0 < \ell \leq \ell_m \Rightarrow sum = N\cdot\ell_m - \sum_{\ell=1}^{\ell_m} k\cdot\ell$

Perfect Hashing | S, $n = |S|$, k, N | $h_k(x) = (kx \bmod N) \bmod n$; $W_i = \{x\in S \mid h_k(x) = i\}$; $b_i = |W_i|$;

$m_i = 2b_i(b_i-1)+1$; k_i so that $h_{k_i}(x) = (k_i x \bmod N) \bmod m_i$ injective on W_i; $s_i = \sum_{j<i} m_j$

Save $x\in S$ at $T[s_i + j]$, $i = (kx \bmod N) \bmod n$, $j = (k_i x \bmod N) \bmod m_i$

Construction time $O(n)$, space $O(n)$, access in $O(1)$

Ackerman | $\alpha(n) = \min\{i \geq 1 \mid A(i,1) > n\}$

$A(0,j) = j+1$

$A(k,j) = A^{(j+1)}(k-1, j)$ for $k \geq 1$

$A^{i+1}(k,j) = A(k, A^i(k,j))$

$A^1(k,j) = A(k,j)$

Bin packing | $NF(I) \leq 2OPT(I)$ | $FF(I) \leq \lceil 17/10\, OPT(I)\rceil$ | $BF(I) = OPT(I)$

NF: $O(n)$; FF: $O(n^2) \longrightarrow O(n\log n) \leftarrow O(n^2)$

$FFD(I) \leq (4OPT(I)+1)/3$

KMP

a	b	a	b	a	c	
-1	0	0	1	2	3	0

$O(n+m)$

Dijkstra | $O(n\log n + m)$ FibHeap

$O(n(T_{Ins} + T_{Empty} + T_{DelMin}) + m\,T_{DecKey} + m + n)$

Bellman-Ford | $O(n^2 + n\cdot m)$ | $Z[v] > n \Rightarrow$ negative cycle | acyclic graph | U = list ord. by $num(v)$

Kruskal | $O(m\cdot\alpha(n) + m + m\cdot\log n)$ | m - Edges | n - nodes

$\alpha(n)$ - Ackermaninverse, $m\cdot\alpha(n)$ from $O(m)$ - Union-Find, $O(n)$ - make-Set

L = list of E sorted by cost | add e to A if no cycle and minimal

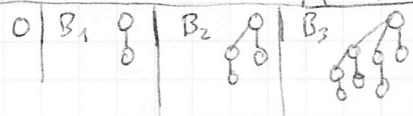
Prim | $O(n\log n + m)$

A is one tree; key is min cost from A to v.

Q = Fib-Heap

	List	Heap	Bin-Q	Fib.Hp
insert	$O(1)$	$O(\log n)$	$\log n$	1
min	$O(n)$	1	$\log n$	1
del-min	$O(n)$	$\log n$	$\log n$	$\log n^*$
meld	$O(1)$	$n/m\log n$	$\log n$	1
dec.key	$O(1)$	$\log n$	$\log n$	1^*

BinTree B_0 B_1 B_2 B_3



B_n has 2^n nodes

B_n height n

root of B_n order n

$\binom{n}{i}$ nodes with height i in B_n

Bin. Queue | n = number of keys

$B_i \in Q \Leftrightarrow (n)_2[i] = 1$

Parent	
entry	degree
child	sibling

$Q.root = null$

$Q.insert(e)$: $B_0.entry = e$, $Q.meld(B_0)$.

$Q.deletemin()$: remove min, invert children, meld.

$Q.dec.Key(v,k)$: $v.entry.key = k$ move $v.entry$ up.

Fib.Heap | Q.min | Q.rootlist | Q.size

left	Parent		right
	entry	degree	
	child	mark	

Q.init(): Q.rootlist = Q.min = null.

Q.meld(K): ~~link~~ unite Q.rootlist + K.rootlist, update Q.min.

Q.insert(e): Q' with e, Q.meld(Q').

Q.deletemin(): remove Q.min from Q.rootlist, add Q.min.childlist in Q.rootlist, Q.consolidate().

Q.consolidate(): $|A| = 2\log_2 n$, link of same degree, update Q.min.

rang(v) = grad, rang(Q) = max grad, Q.size = n.

Q.decreasekey(v,k): v.key = k, update Q.min, cascading cuts if k < parent, mark on child remove. update degree for parent

V becomes child $\Rightarrow$ v.mark = false | v loses child $\Rightarrow$ v.mark = true | v loses 2nd. child $\Rightarrow$ cut(v)

Disjoint-set | e.makeset(): e.parent = e, e.size = 1. Link(x,y): smaller x becomes child, size of parent increased.

find-set in $O(\log n)$ | compression + union by rank $\Rightarrow \Theta(m\cdot\alpha(n))$

m operations, f - find-set, n - make-set $\to$ at most n-1-unions: Link by rank in $O(n + f\log n)$, find-set with compression if $f < n$: $\Theta(n + f\log n)$ else $\Theta(f\log_{1+f/n} n)$

Closest Pair | $S = \{p_i\}$;

Sort S by x-axis inc.

minDist(S): $S_\ell = \{p_a, p_b, p_c, p_d\}$; $S_r = \{p_e, p_f, p_g, p_h\}$

$d_\ell = minDist(S_\ell)$; $d_r = minDist(S_r)$

bound $d = \min(d_\ell, d_r)$; points within bound: (p_c, p_f, p_g)

sort by y-axis, test pairs less than 16 vertical diff.

return the min d. | $\Theta(n\cdot\log n)$

minDist(S_ℓ)...

Interval Scheduling | (S, f) = (starts, finishing times)

f-sorted inc. Pick interval with earliest f-time

$\Theta(n)$ [+ sort by f in $\Theta(n\log n)$]

ReportCuts | $O(n\log n + k)$ k = #Cuts

ReportCuts(S) = ReportCuts(S_1) $\cup$ RC(S_2) $\cup$ M

RC(S_1): $L(S_1) = \{A \mid$ only left-end A in $S_1\}$

$R(S_1) = \{A \mid$ only right-end A in $S_1\}$

RC(S_2) $V(S_1) = \{a \mid a$ is vertical in $S_1\}$

$L(S) = (L(S_1) \setminus R(S_2)) \cup L(S_2)$

$R(S) = (R(S_2) \setminus L(S_1)) \cup R(S_1)$; $V(S) = V(S_1) \cup V(S_2)$

L, R sorted by increasing y-pos | V sorted by inc. lower and points

$M = M_1 \cup M_2$ | $Cut(P,p)$: $p.\underline{y} \leq P.y \leq p.\overline{y}$

$M_1 = \{(P,p) \mid P \in R(S_2) \setminus L(S_1) \wedge p \in V(S_1) \wedge Cut(P,p)\}$

$M_2 = \{(P,p) \mid P \in L(S_1) \setminus R(S_2) \wedge p \in V(S_2) \wedge Cut(P,p)\}$