Constraint Satisfaction Problems

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Exercise Sheet 2

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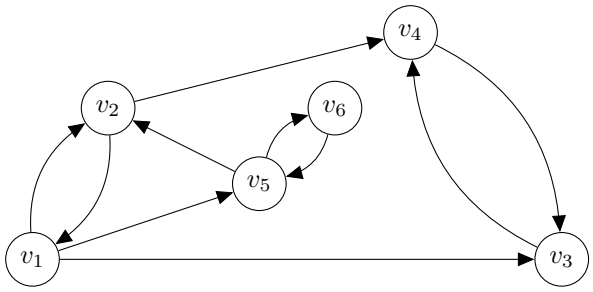
Exercise 2.1 (1+2 Points)

Let $G = \langle V, A \rangle$ be a directed graph given by the following figure.



(a) **Strongly connected components** of a graph are the maximal strongly connected subgraphs.
List the strongly connected components of G.

(b) Let $G' = \langle V, E \rangle$ be the undirected simple graph obtained from G by setting $\{v, v'\} \in E$ if and only if $(v, v') \in A \vee (v', v) \in A$.
List all cliques of G'.
Hint: The definition allows for Cliques of size one.

Exercise 2.2 (2 Points)

A group of coworkers W is to be moved into new offices in two different buildings. Some workers dislike each other and moving any such two people into the same building leads to a conflict.

> **Decision problem**: Given a finite set $W := \{w_1, \ldots, w_n\}$ of workers and a list of sets $\{w, w'\}$, with $w, w' \in W$ that indicate that w and w' dislike each other, is it possible to distribute the workers to two buildings such that no conflict arises?

Describe a polynomial time algorithm for this decision problem (and thus proof that it is solvable in polynomial time).

Exercise 2.3 (2+3 Points)

Given an undirected graph $G = (V, E)$ and a positive integer $k \leq |V|$, we define the following decision problems:

Clique: Is there a subset $V' \subseteq V$ such that $|V'| \geq k$ and for each pair $v_1, v_2 \in V'$, $v_1 \neq v_2$ it holds $(v_1, v_2) \in E$?

Vertex Cover: Is there a subset $V' \subseteq V$ such that $|V'| \leq k$ and for each edge $(v_1, v_2) \in E$ at least one of v_1 and v_2 belongs to V'?

Dominating Set: Is there a subset $V' \subseteq V$ such that $|V'| \leq k$ and such that every vertex $v \in V \setminus V'$ is connected to a vertex in V'?

Proof the NP-completeness of

(a) Vertex Cover by reduction from Clique,

(b) Dominating Set by reduction from Vertex Cover.