CSP Exercise 01 Solution

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Exercise 1.1

5	3	6	4	7	2	8	1	9
4	2	1	6	9	8	5	7	3
7	8	9	1	5	3	4	6	2
8	6	3	9	2	4	1	5	7
1	7	5	8	3	6	9	2	4
2	9	4	5	1	7	6	3	8
3	1	8	2	4	5	7	9	6
9	4	7	3	6	1	2	8	5
6	5	2	7	8	9	3	4	1

I have applied following strategies to solve the puzzle:

- look for most contrained blocks, rows and columns
- look for values with most resolved positions
- annotate cells with consistent values
- look for inconsistent connections between cell annotations

Exercise 1.2

(a) $R_{x,y} \bowtie S_{y,z} = \{(a, b, a), (a, b, c)\}$

(b) $\sigma_{z=c}(R_{x,y} \bowtie S_{y,z}) = \{(a, b, c)\}$

(c) $\pi_x(R_{x,y}) = \{(a)\}$

(d) $R_{x,y} \circ S_{y,z} = \{(a, a), (a, c)\}$

Exercise 1.3

(a) By definition holds

$$R \circ (S \cup T) = \{(x, z) \in X^2 \mid \exists y \in X : (x, y) \in R \text{ and } (y, z) \in (S \cup T)\} \tag{1}$$
$$R \circ S = \{(x, z) \in X^2 \mid \exists y \in X : (x, y) \in R \text{ and } (y, z) \in S\} \tag{2}$$
$$R \circ T = \{(x, z) \in X^2 \mid \exists y \in X : (x, y) \in R \text{ and } (y, z) \in T\} \tag{3}$$

The union of (2) and (3) gives us the set of all $(x, y) \in R$ with either $(y, z) \in S$ or $(y, z) \in T$, from which follows that $(y, z) \in (S \cup T)$, making it equal to (1). □

(b) By definition holds

$$-R = \{(x,y) \in X^2 \mid (x,y) \notin R\} \tag{1}$$
$$(-R)^{-1} = \{(y,x) \in X^2 \mid (x,y) \in -R\} \tag{2}$$
$$-(R^{-1}) = \{(x,y) \in X^2 \mid (x,y) \notin R^{-1}\} \tag{3}$$
$$-(R^{-1}) = \{(x,y) \in X^2 \mid (y,x) \notin R\} \tag{4}$$

We've obtained (4) by applying the converse definition on (3). From the definition of $-R$ (1) follows that $(x,y) \in -R$ iff $(x,y) \notin R$, and therefore we see that (2) and (4) define equal sets. □

(c) By definition holds

$$(R \circ S)^{-1} = \{(z,x) \in X^2 \mid \exists y \in X : (x,y) \in R \text{ and } (y,z) \in S\} \tag{1}$$
$$S^{-1} \circ R^{-1} = \{(x,z) \in X^2 \mid \exists y \in X : (x,y) \in S^{-1} \text{ and } (y,z) \in R^{-1}\} \tag{2}$$
$$S^{-1} \circ R^{-1} = \{(x,z) \in X^2 \mid \exists y \in X : (y,x) \in S \text{ and } (z,y) \in R\} \tag{3}$$

We've obtained (3) by applying the converse definition on (2). After some variable juggling we see that (1) and (3) define equal sets. □

(d) By definition holds

$$R \circ S = \{(x,z) \in X^2 \mid \exists y \in X : (x,y) \in R \text{ and } (y,z) \in S\} \tag{1}$$
$$(R \circ S) \cap T^{-1} = (R \circ S) \cap \{(y,x) \in X^2 \mid (x,y) \in T\} \tag{2}$$
$$(R \circ S) \cap T^{-1} = \{(x,z) \in X^2 \mid \exists y \in X : (x,y) \in R \text{ and } (y,z) \in S \text{ and } (z,x) \in T\} \tag{3}$$
$$(S \circ T) \cap R^{-1} = \{(x,z) \in X^2 \mid \exists y \in X : (x,y) \in S \text{ and } (y,z) \in T \text{ and } (z,x) \in R\} \tag{4}$$

We've obtained (3) by directly applying intersection within the set comprehension of (2). It follows that

$$(R \circ S) \cap T^{-1} = \emptyset \iff \forall x,y,z \in X : (x,y) \notin R \text{ or } (y,z) \notin S \text{ or } (z,x) \notin T \tag{5}$$
$$\text{and } (S \circ T) \cap R^{-1} = \emptyset \iff \forall x,y,z \in X : (x,y) \notin S \text{ or } (y,z) \notin T \text{ or } (z,x) \notin R \tag{6}$$

As we can see, the intersections form a ring-like relationship between the sets' tuples $(..., R, S, T, R, ...)$. After some variable reordering, we see that both intersections are empty iff there is no ring-like relationship between any tuples of the three sets. □