Algorithmic Verification of Reactive Systems

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Seminar: Automata Constructions in Model Checking

Motivation

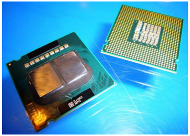
Model Checking 1/2

$$\mathcal{M} \models \varphi$$

Given a program P and specification φ:

does every run of P satisfy φ?

Motivation
Industry



Motivation

Industry

"It is dark."

"It is dark."
"It is *always* dark."

"It is dark."
"It is *always* dark."
"It is *currently* dark."

"It is dark."
"It is *always* dark."
"It is *currently* dark."
"It will *necessarily* be dark."

"It is dark."

"It is *always* dark."

"It is *currently* dark."

"It will *necessarily* be dark."

"It is dark *until* someone puts the light on."

Linear Temporal Logic

Natural language 2/2

It is dark until there is light

Linear Temporal Logic

Natural language 2/2

It is dark	until	there is light
p_0	$\mathcal{U}$	p_1

Syntax

Let $\mathcal{P}$ be the countable set of *atomic propositions*, LTL-formulae φ are defined using following productions:

$$\varphi ::= p \in \mathcal{P} \,|\, \neg\varphi \,|\, \varphi \vee \varphi \,|\, \mathcal{X}\varphi \,|\, \varphi\mathcal{U}\varphi$$

- $\neg, \vee$ corresponds to the Boolean *negation* and *disjunction*.
- $\mathcal{X}$ reads *next*.
- $\mathcal{U}$ reads *until*.

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Derived connectives

Let φ and ψ be formulae:

- $\top \equiv p \vee \neg p$ for $p \in \mathcal{P}$
- $\bot \equiv \neg\top$
- $\varphi \wedge \psi \equiv \neg(\neg\varphi \vee \neg\psi)$
- $\varphi \rightarrow \psi \equiv \neg\varphi \vee \psi$
- $\varphi \leftrightarrow \psi \equiv (\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$
- $\Diamond\varphi \equiv \top\mathcal{U}\varphi$
- $\Box\varphi \equiv \neg\Diamond\neg\varphi$

Frame

An LTL-*frame* is a tuple $\mathcal{F} = (S, R)$:

- $S = \{s_i \mid i \in \mathbb{N}_0\}$ is the set of states.
- $R = \{(s_i, s_{i+1}) \mid i \in \mathbb{N}_0\}$ is the accessibility relation.

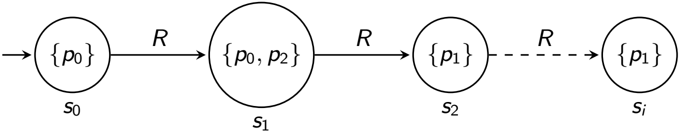
Model

An LTL-*model* is a tuple $\mathcal{M} = (\mathcal{F}, V)$:

- $\mathcal{F}$ is a *frame*.
- $V : S \to 2^{\mathcal{P}}$ is a *valuation function*.
- Intuitively we say $p \in \mathcal{P}$ is *true* at time instant i iff $p \in V(i)$.

Linear Temporal Logic

Model Example



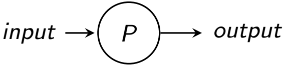
Satisfiability

A model $\mathcal{M} = (\mathcal{F}, V)$ *satisfies* a formula φ at time instant i is denoted by $\mathcal{M}, i \models \varphi$:

- $\mathcal{M}, i \models p$ for $p \in \mathcal{P} \iff p \in V(i)$
- $\mathcal{M}, i \models \neg\varphi \iff$ not $\mathcal{M}, i \models \varphi$
- $\mathcal{M}, i \models \varphi \vee \psi \iff \mathcal{M}, i \models \varphi$ or $\mathcal{M}, i \models \psi$
- $\mathcal{M}, i \models \mathcal{X}\varphi \iff \mathcal{M}, i+1 \models \varphi$
- $\mathcal{M}, i \models \varphi\mathcal{U}\psi \iff \exists k \geq i : \mathcal{M}, k \models \psi$ and $\forall i \leq j < k : \mathcal{M}, j \models \varphi$

Reactive Systems

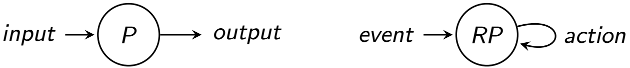
Infinite inputs



(a) Terminating program

Reactive Systems

Infinite inputs

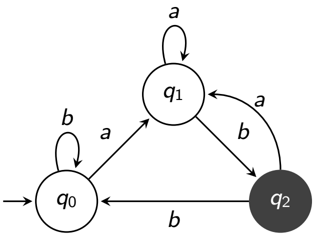


(a) Terminating program

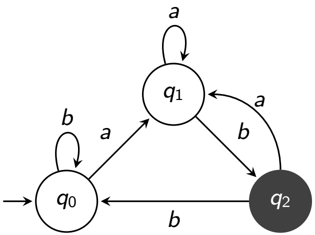
(b) Reactive program

Automata

Example 1/2



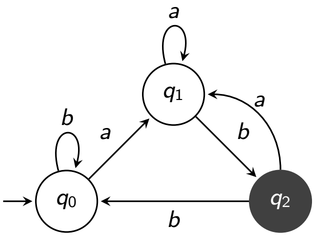
Automata
Example 1/2



$$w_1 = \overline{bbaa} \implies \rho_1 = q_0 q_0 \overline{q_0 q_1 q_1 q_2}$$

Automata

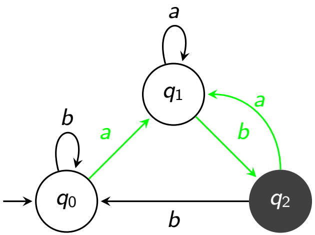
Example 1/2



$$w_1 = \overline{bbaa} \implies \rho_1 = q_0 q_0 \overline{q_0 q_1 q_1 q_2}$$
$$w_2 = bb\overline{ab} \implies \rho_2 = q_0 q_0 \overline{q_1 q_2}$$

Automata

Example 1/2

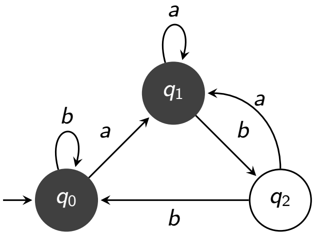


$$w_1 = \overline{bbaa} \implies \rho_1 = q_0 q_0 \overline{q_0 q_1 q_1 q_2}$$
$$w_2 = bb\overline{ab} \implies \rho_2 = q_0 q_0 \overline{q_1 q_2}$$

Accepts all inputs with infinite occurrences of *ab*.

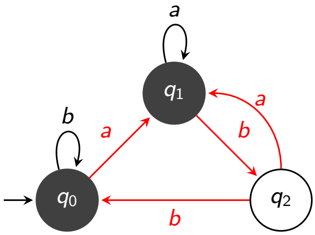
Automata

Example 2/2 (Complement)



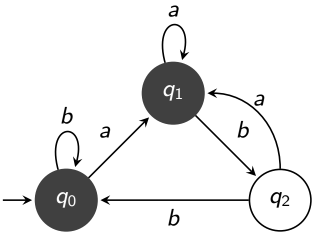
Automata

Example 2/2 (Complement)

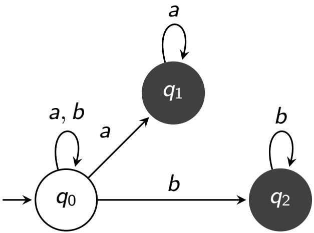


Automata

Example 2/2 (Complement)



(a) Complement automaton ✗



(b) Complement automaton ✓

Accepts all inputs with finite many *ab*.

Automaton

A non-deterministic Büchi automaton is a tuple $\mathcal{A} = (\Sigma, S, S_0, \Delta, F)$ with:

- Σ is a finite *alphabet*.
- S is a finite set of *states*.
- $S_0 \subseteq S$ is the set of *initial states*.
- $\Delta : S \times \Sigma \times S$ is a *transition relation*.
- $F \subseteq S$ is the set of *accepting states*.

Run

Let $\mathcal{A} = (\Sigma, S, S_0, \Delta, F)$ be an automaton:

- A run ρ of $\mathcal{A}$ on an infinite word $w = a_0, a_1, \ldots$ is a sequence $\rho = s_0, s_1, \ldots$:
  - $s_0 \in S_0$.
  - $(s_i, a_i, s_{i+1}) \in \Delta$, for all $i \geq 0$.
- Alternative view of a run ρ as a function $\rho : \mathbb{N}_0 \rightarrow S$, with $\rho(i) = s_i$.

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$$w_1 = \overline{bbaa} \implies \rho_1 = q_0 q_0 \overline{q_0 q_1 q_1 q_2}$$

$$w_2 = bb\overline{ab} \implies \rho_2 = q_0 q_0 \overline{q_1 q_2}$$

Infinite occurrences

Let $\mathcal{A} = (\Sigma, S, S_0, \Delta, F)$ be an automaton and let ρ be a run of $\mathcal{A}$:

- $\exists^\omega$ denotes the existential quantifier for *infinitely* many occurrences.
- $\mathit{inf}(\rho) = \{s \in S \mid \exists^\omega n \in \mathbb{N}_0 : \rho(n) = s\}$.

Acceptance

Let $\mathcal{A} = (\Sigma, S, S_0, \Delta, F)$ be an automaton and let ρ be a run of $\mathcal{A}$:

- ρ is *accepting* iff $\mathit{inf}(\rho) \cap F \neq \emptyset$.
- $\mathcal{A}$ *accepts* an input word w iff there exists a run ρ of $\mathcal{A}$ on w, such that ρ is accepting.

Recognised language

Let $\mathcal{A} = (\Sigma, S, S_0, \Delta, F)$ be an automaton:

- $L_\omega(\mathcal{A}) = \{w \in \Sigma^\omega \mid \mathcal{A} \text{ accepts } w\}$, we say $\mathcal{A}$ recognises language $L_\omega(\mathcal{A})$.
- Language L is *Büchi-recognisable* iff there is an automaton $\mathcal{A}$ with $L = L_\omega(\mathcal{A})$.

Generalised Automata

Generalised automaton

A *generalised Büchi automaton* is a tuple $\mathcal{A}_G = (\Sigma, S, S_0, \Delta, \{F_i\}_{i<k})$:

- $\{F_i\}$ is a finite set of sets for a given k.
- Each $F_i \subseteq S$ is a finite set of accepting states.

Acceptance

Let $\mathcal{A}_G = (\Sigma, S, S_0, \Delta, \{F_i\}_{i<k})$ be a generalised automaton and let ρ be a run of $\mathcal{A}_G$:

- ρ is *accepting* iff $\forall i < k : \mathit{inf}(\rho) \cap F_i \neq \emptyset$.
- $\mathcal{A}_G$ *accepts* an input word w iff there exists a run ρ of $\mathcal{A}_G$ on w, such that ρ is accepting.

Proposition

Let $\mathcal{A}_G = (\Sigma, S, S_0, \Delta, \{F_i\}_{i<k})$ be a generalised automaton and let $\mathcal{A}_i = (\Sigma, S, S_0, \Delta, F_i)$ be non-deterministic automata:

$$L_\omega(\mathcal{A}_G) = \bigcap_{i<k} L_\omega(\mathcal{A}_i)$$

Model $\mathcal{M}_\varphi$ for formula φ

$\Downarrow$

Closure $CL(\varphi)$ of φ

$\Downarrow$

Automaton $\mathcal{A}_\varphi$ for φ

Model $\mathcal{M}_\varphi$ for formula φ

$\Downarrow$

Closure $CL(\varphi)$ of φ

$\Downarrow$

Automaton $\mathcal{A}_\varphi$ for φ

On-the-fly method à la Gerth et al.

Automata Construction

System automata 1/2

Program

Given a program $P = (S_P, s_0, R, V)$:

- S is the set of possible states.
- s_0 is the initial state.
- $R : S \times \mathcal{P} \times S$ is the transition relation.
- $V : S \to 2^{\mathcal{P}}$ is a valuation function.

A *computation* of P is a run $\rho = (V(s_0), V(s_1), ...)$.

System automaton

We construct automaton $\mathcal{A}_P = (\Sigma, S, S_0, \Delta, F)$ for program P:

- $\Sigma = 2^{\mathcal{P}}$
- $S = S_P$
- $\Delta = \{(s, V(s), s') \mid \exists a \in \mathcal{P} : (s, a, s') \in R\}$
- $S_0 = \{s_0\}$
- $F = S$

Proposition

Let $\mathcal{A}_P = (\Sigma, S, S_0, \Delta, F)$, note that $F = S$, it follows:

$$L_\omega(\mathcal{A}_P) = \{\rho \mid \rho \text{ is a run of } \mathcal{A}_P\}$$

Verification

Given a program P and specification φ:

does every run of P satisfy φ?

Verification

Given a program P and specification φ:

$$L_\omega(\mathcal{A}_P) \subseteq L_\omega(\mathcal{A}_\varphi)$$

Verification

Given a program P and specification φ:

$$L_\omega(\mathcal{A}_P) \cap L_\omega(\mathcal{A}_{\neg\varphi}) = \emptyset$$

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