A Unified Model for Evaluating the Safety Integrity Level of Safety Instrumented Systems

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Key Words: safety integrity levels, safety instrumented systems standards, IEC 61508, ANSI/ISA 84.00.01-2004, PFD_{avg}, probability of accident, process demand models, time to periodic inspection, Markov models, risk reduction factor

SUMMARY & CONCLUSIONS

This paper proposes a new figure of merit (FOM) for evaluating safety integrity levels (SIL) for safety instrumented systems (SIS). Currently, SIL ratings are based on two separate tables – one for low process demands and one for high process demands. The proposed FOM, probability of an accident as a function of time, PAC(t), unifies the two separate tables into a single table and extends the concept of risk reduction factor (RRF), which is currently only defined for low demand applications, to high demand applications as well. Using PAC(t) as the new FOM explicitly includes the process demand rate in the model and therefore, permits the effects of different demand rates on the safety performance of a specific SIS to be quantified. The model also allows for the inclusion of diagnostic coverage and on-line repair so that the effects of these parameters can also be quantified. Finally, using PAC(t), the maximum time of periodic inspection (T_I) permitted before the SIS moves to a lower SIL rating can be easily calculated. A number of examples illustrate the application and usefulness of PAC(t) as the defining FOM for SIL evaluation.

1 INTRODUCTION

Safety instrumented systems are automatic systems designed for the purpose of taking an action to avoid an accident or minimize its consequences. An SIS typically has two distinct failure modes. The safe failure mode erroneously intervenes in a correctly operating process. The dangerous failure mode, often called failed dangerous (FD), occurs when the SIS loses the ability to intervene in the process in the event that the process requires intervention (PRI). Both modes may be detectable (by self-diagnostics) or undetectable. Undetectable FD modes are normally found and repaired during periodic inspection and maintenance typically scheduled at regular deterministic intervals of length T_I .

From a safety viewpoint, the FD states, both detected (FDD) and undetected (FDU), are of greatest concern. While the FD state itself does not cause an accident, it is the window of opportunity for an accident to occur should the PRI while the system is in an FD state. The consequences of an accident vary by process and can range from minor to extensive damage, minor to serious injury and/or loss of life. Clearly, the more severe the consequences of an accident, the more

emphasis need to be placed on ensuring a suitably small probability that the SIS is in the state FD.

Safety Integrity, defined in two international, performance-based standards [1, 2], is the probability that a SIS successfully fulfills its intended safety function. The standards further define four discrete SIL. The lowest level (corresponding to minimal consequences in the event of an accident) allows for a higher probability of being in the FD state, while the highest level (corresponding to the most severe consequences) requires the least probability of being in the FD state.

This concept of SIL would be easy to understand and use if there were a single definition for SIL and a single definition for the FOM to be computed and then compared to the various SIL criteria. However, the safety achieved by the SIS depends not merely on *its* performance characteristics but also on the rate at which the PRI, i.e., the process demand, λ_p . Consequently, there are two distinctly different definitions for SIL each with a distinctly different FOM to be computed. One definition is used when process demand is low, the other, when process demand is high. According to [1], demand is low if the rate of periodic inspection is two or more times the demand rate. So, for example, if T_1 were 2 years and therefore the rate of periodic inspections were 1 inspection/ (2 years) = 2.5 inspections /(5 years), and the demand rate, λ_p , were 1 demand/(5 years), the demand would be considered low and the FOM would be computed one way. However, for the same SIS and same process, if T_I were changed to 4 years and, therefore, the periodic inspection rate were changed to 1 inspection/(4 years) = 1.25 inspections/(5 years), the demand rate of 1/(5 years) would be considered high and the FOM would be computed a different way. Furthermore, for a particular SIS with a fixed rate of periodic inspection, say 1/(3 years), consistent with low demand, the calculated FOM would be the same regardless of whether the demand rate were $1/(10 \text{ years})$ or $1/(25 \text{ years})$ even though one would instinctively feel that the probability of an accident (PAC) would be less with the smaller demand rate of 1/(25 years) than the larger demand rate of 1/(10 years).

Publications addressing SIL evaluation tend to limit themselves to either low demand or high demand and do not include the demand rate explicitly in the computation of the SIL FOM. One work that separately addresses both high and

1-4244-1461-X/08/\$25.00 ©2008 IEEE

low demand and includes the demand rate [3] notes that, between truly low and truly high demand, there exists a gray area where SIL evaluation under either SIL specification does not really capture the relative safety issues involved.

It would be useful if a single FOM existed that incorporated all of the relevant parameters and was applicable for any demand rate. This paper:

- reviews and critically assesses the current definitions for SIL;
- proposes a FOM consistent with the objectives of SIL that incorporates all relevant parameters;
- shows how the proposed FOM unites the two individual tables into a single consistent SIL table;
- relates the new FOM to a time dependent risk reduction factor, RRF(t), and shows that RRF(t) can be extended to include high demand as well as low demand;
- compares the results computed using the proposed FOM with those obtained using the current FOM;
- demonstrates how the results can be used to determine the T_I required to achieve a particular SIL level; and,
- demonstrates how the effect of diagnostic coverage and on-line repair rates on SIS safety performance can be quantified.

2 NOTATION

 λ_{P} rate at which the process moves from safe progression to requiring intervention; "failure rate" of process in "failures"/hour (often called "demand rate")

Λ transition matrix for the Markov model

- μ_{DD} on line-repair rate for FDD failures in repairs/hr; 1/average repair time
- $\pi(t)$ row vector whose ith entry represents the probability of being in the ith state of the Markov model at time t
- $\pi_i(t)$ ith entry of the row vector $\pi(t)$
- $\pi(0)$ the initial distribution of states in the Markov model

3 BACKGROUND

Based on information from [1, 2], Tables 1 and 2 below summarize the SIL definitions for low and high demand rates, respectively.

SIL	$FOM = PFD_{avg}$	$RRF = 1/PFD_{avg}$
	10^{-1} 10	(10, 100]
	10^{-7}	(100, 1,000]
		(1,000, 10,000]
		(10,000, 100,000]

Table 1. SIL specifications for low demand applications.

Table 2. SIL specifications for high demand applications.

Recall that a demand is considered low if the rate of periodic inspection is two or more times the demand rate. Periodic inspection can be a very costly proposition, and, consequently, periodic inspection is rarely performed more frequently than once in 6 months and is often performed much less frequently, i.e., at much longer intervals. Thus, as a practical matter, the low demand table generally applies to demand rates of 1/year or less. In Table 1, the FOM is an average probability and hence is dimensionless. PFD_{avg} is computed as

$$
PFDayg = \frac{1}{T_I} \int_{0}^{T_I} PFD(t) dt
$$
 (1)

where T_I is the time to periodic inspection and maintenance. Thus, the FOM of Table 1 implicitly includes the parameter Γ_{L} . However, it does not include, implicitly or explicitly, the parameter λ_p . Thus, the same FOM will be computed for a given SIS regardless of the demand rate of the application.

The standards in [1, 2] allow, within some guidelines, considerable latitude in choosing a model and method for computing PFD_{avg} or $\text{PFD}(t)$ which is then averaged. Column 3 of Table 1 is the Risk Reduction Factor (RRF), defined as the inverse of PFD_{avg} . RRF captures the information contained in the small probabilities of PFD_{avg} in a way that is

more intuitively understood. Subject to several assumptions including the assumption that the event FD and the event PRI are independent, the RRF approximates the ratio of average probability of an accident (PAC_{avg}) without the SIS to the PAC_{avg} with the SIS. Thus a RRF of 10 is interpreted to mean that the SIS reduces the PAC_{avg} by a factor of 10 compared to the process running without the SIS.

It should be noted that PFD_{avg} computed at T_I is approximately half the instantaneous value computed by $PFD(T_1)$. Using an average quantity for the FOM that determines the low demand SIL guards against over-design for systems used in low demand applications. However, the user of the SIS needs to be aware that $\text{PFD}(t)$ exceeds the PFD_{avg} for about half of the periodic inspection interval. This means that if the RRF, computed from PFD_{avg} , is 10, the SIS is reducing the PFD(t) by 10 or more over the first half of the interval $[0, T_I]$ but by less than 10 on the second half of the interval.

Table 2 is intended for use in high demand applications. It includes the most severe case of continuous demand. In this case, any failure of the SIS leads to an accident. Thus the frequency of SIS failure in the state FD, i.e., λ_{D} , forms the basis of the FOM whose dimensions are inverse time. Again, the demand rate, λ_p , in not included in the FOM either implicitly or explicitly. Thus, a given SIS will have the same SIL regardless of whether the application to which it is applied has a continuous process demand rate, i.e., $\lambda_p = \infty$, or a process demand rate of $\lambda_p = 1/(11 \text{ months}).$

The use of λ_D as the FOM assumes that the PFD(t) is an exponential function and therefore a constant λ_D applies. This may be reasonable for some SIS. However, for redundant systems and systems with diagnostic coverage [4, 5] that allow for on-line repair from the FD state, the PFD(t) is not likely to be exponential and therefore not represented by a single constant λ_{D} . Furthermore, even if one were to accept a constant λ_D as an approximation of a more complex SIS failure rate, computing the constant λ_D to approximate the SIS failure behavior may be quite complicated, assuming it is even possible to do so. The RRF is not defined for high demand.

4 PROPOSED NEW FOM FOR EVALUATING SIL

Figure 1a shows a simplified Markov model that can be used to evaluate the two current SIL FOM. These FOM concentrate on the FD state of the SIS whether through the failure rate, λ_D , or the related PFD_{avg}. The current FOM focus on the window of opportunity for an accident to occur, whether or not being in this state, actually leads to an accident. The proposed new FOM focuses instead on the probability of an accident actually occurring. Consequently it requires a model that explicitly includes the demand rate, λ_p , and shows the state AC, the state where an accident actually occurs because the SIS is in a state of FD and the PRI. Figure 1b is an expanded version of the Markov model in Figure 1a. The additional state is the state of an accident (AC). The proposed FOM for evaluating SIL is PAC(t). This quantity was previously investigated in [6] although there is was referred to as PFDPRI(t) and that work did not relate the calculated quantity to SIL in any way.

Figure 1. Simplified Markov models to evaluate SIL.

4.1 *Relationship of Proposed FOM to SIL Tables 1 and 2*

In order to explore the relationships between the two current FOM and the proposed FOM with respect to the SIL in Tables 1 and 2, begin by considering the worst case high demand scenario, i.e., the case of continuous demand. In this case, Table 2 applies, the FOM is λ_{D} , and the probability of being in the FD state is synonymous with the occurrence of an accident since any failure of the SIS causes an accident under the conditions of continuous demand. Figure 1a serves as a simple Markov model and the new FOM, PAC(t) would be calculated as

$$
PAC(t) = PFD(t) = 1 - e^{-\lambda Dt}.
$$
 (2)

Figure 1b is a more general Markov model that allows for the possibility that the demand is not continuous. The Markov model is completely characterized by its transition matrix, Λ , and its behavior is governed by the matrix differential equation

$$
\frac{d}{dt}\pi(t) = \pi(t)\Lambda.
$$
 (3)

The solution to (3) is

$$
\pi(t) = \pi(0)e^{-\Lambda t}.
$$
 (4)

For Figure 1b, Λ is given by

$$
\Lambda = \begin{bmatrix} -\lambda D & \lambda D & 0 \\ 0 & -\lambda p & \lambda p \\ 0 & 0 & 0 \end{bmatrix} . \tag{5}
$$

Assuming that the SIS begins operation in the state OK, i.e., $\pi(0) = [1 \ 0 \ 0]$, the closed-form solutions for the state probabilities in Figure 1b are

$$
\pi_1(t) = P(OK) = e^{-\lambda Dt},\tag{6}
$$

$$
\pi_2(t) = PDF(t) = \frac{-1}{1 - (\lambda p / \lambda D)} e^{-\lambda D t} + \frac{1}{1 - (\lambda p / \lambda D)} e^{-\lambda p t}, (7)
$$

$$
\pi_3(t) = PAC(t) = 1 + \frac{(\lambda p/\lambda D)}{1 - (\lambda p/\lambda D)} e^{-\lambda Dt} - \frac{1}{1 - (\lambda p/\lambda D)} e^{-\lambda pt}.
$$
 (8)

Note that these solutions apply provided that $t > 0$, $\lambda_p > 0$, $\lambda_p > 0$ 0, and $\lambda_{\text{D}} \neq \lambda_{\text{p}}$. If $\lambda_{\text{D}} = \lambda_{\text{p}}$, then a Jordon form solution is necessary and is not presented here due to space limitations and the fact that the equality is very unlikely to occur.

If $\lambda_p = \infty$, i.e., if the demand is continuous, then PAC(t) as given in (8) reduces to PAC(t) as given in (2). Thus the model of Figure 1b is more general in that it allows for the demand rate to be explicitly modeled. It also shows that the event of the SIS reaching a state of FD and the event of PRI are not independent in general. If they were, then PAC(t) would be the product of PFD(t)*PRI(t). Assuming that λ_p is constant, then this would mean that (8) would factor into the product of $(1-exp(-\lambda_D t))*(1-exp(-\lambda_D t))$ which it clearly does not. It can be shown (though it is not included in this paper) that the two aforementioned events are approximately independent only under the conditions of very low demand.

Consider how PAC(t) behaves in the case of continuous demand for SIL 4. In this case, $10^{-9} \le \lambda_D < 10^{-8}$ and PAC(t) for $\lambda_D = 10^{-8}$ (the upper bound of SIL 4) is shown in Figure 2 as the dashed line. Clearly, the PAC increases with time and there must be some upper limit on the PAC that is tolerable. This tolerable upper limit on PAC links Tables 1 and 2. In Table 1, for the case of SIL 4, what does the requirement that PFD_{avg} < 10⁻⁴ means in terms of PAC(t)? The worst case low demand scenario is a demand rate of 1/year, implying $T_1 = 0.5$ years. Assuming that the events FD and PRI are independent, and recalling that PFD(t) \approx 2 * PFD_{avg}, the PAC(T_I) is given by

$$
PAC(T_I) \approx PFD(T_I) * PRI(T_I)
$$

\n
$$
\approx 2 * PFD_{avg} * \lambda_p * T_I
$$

\n
$$
\approx 2 * 10^{-4} * \frac{1}{year} * 0.5 years = 10^{-4}.
$$
 (9)

Therefore, the upper bound on the SIL 4 in Table 1 also represents the upper bound on PAC(t) for the worst case low demand scenario covered by Table 1. From the point of view of Table 2, the upper bound on the PAC(t), shown in Figure 2 as the solid line, is reach at approximately 10,000 hours or approximately 1 year of usage under continuous demand, and corresponds to the maximum value of PAC that still qualifies

Figure 2. PAC(t) and max PAC for SIL 4 Table 1.

as SIL 4.

This general relationship holds for the other SIL as well. The upper limit on PAC(t) for a given SIL is equal to the maximum allowable FOM value for that SIL in Table 1 and corresponds to the PAC(t) computed from the upper limit on the same SIL in Table 2 at $t = 10,000$ hours. Thus, the Tables 1 and 2 could be merged if the FOM were PAC(t) and the boundaries of the SIL remained as they are in Table 1, though now applied to PAC(t) rather than $\text{PFD}_\text{avg.}$ Additionally, if RRF were replaced by $RRF(t)=1/PAC(t)$, $RRF(t)$ would represent the risk reduction achieved at t and the minimum risk reduction achieved on the time interval [0, t]. The proposed single SIL table is illustrated in Table 3.

4.2 *Practical Issues in Computing PAC(t)*

It may appear from (8) that the proposed FOM, PAC(t), will be too difficult to compute in general and that this will limit its usefulness. This is not a cause for concern. While it is true that writing general closed-form solutions like (6)-(8) in terms of system parameters for an arbitrarily complex model is a difficult, if not intractable, problem, it is rarely necessary to write such a general solution. Normally, all information of interest is easily computed by substituting a range of values for t into (4). Indeed, the graphs produced in the next section were so constructed using less than a dozen lines of MATLAB code. If any FOM present difficulty in non-approximate computation for any but the simplest of models, it is the current FOM.

5 EXAMPLES APPLYING THE PROPOSED FOM

The Markov model of Figure 1b is the minimum required to compute PAC(t). However, the SIS model to the left of the state AC could be replaced by an SIS model of any level of detail and complexity as long as all states representing the SIS in a state of FD are permitted to transit to state AC via a transition rate equal to λ_p .

For the examples presented here, the model shown in Figure 3a, which does not include λ_p , is used to compute PFDavg. The model shown in Figure 3b, which explicitly includes λ_p , is used to compute PAC(t). These models include the possibility of self-diagnostics represented by C, the diagnostic coverage, and of on-line repair represented by μ_{DD} , the repair rate for dangerous detected failures. (It could be argued that the models in Figure 1 can include diagnostic coverage by replacing λ_D by $\lambda_{DU} = \lambda_D^*(1-C)$. However, the effects of the on-line repair rate are not included in the models of Figure 1 and using λ_{DU} instead of λ_{D} is not permitted with respect to the SIL in Table 2.)

Using the models of Figure 3, it is possible not only to examine the SIS behavior under different demand conditions, but also to see the effects of C and μ_{DD} under fixed demand conditions. If the SIS has no diagnostic coverage, the same model applies but with C set to 0. Several of these possibilities are explored in the examples below. Table 4 indicates the parameter values used in each of the examples.

Figure 3. Markov models used for the examples.

Parameter	Ex. 1	Ex. 2	Ex.3
√∿p	varies	1 /week	$/$ month
Λŋ		$5*10^{-6}$	$5*10^{-6}$
	failures/hr	failures/hr	failures/hr
	19	varies	0.5
$\mu_{\rm DD}$	/8 hr	$/8$ hr	varies

Table 4. Parameter values for the examples.

5.1 *Example 1*

In this example, λ_{D} , C, and μ_{DD} are held fixed at the values in Table 4 while λ_p is varied over values including both high and low demand. The results are shown in Figure 4. Begin by comparing the results under the proposed FOM, PAC(t), with the results from the current applicable FOM, PFD_{avg} from Table 1. In Figure 4, PFD_{avg} is shown as the dashed line and is computed from Equation (1) for values of T_I between 0 and 10 years. According to the PFD_{avg} information in Figure 4, the SIS would be classified as SIL 1 provided its T_1 were about 2.3 years or less because $T_1 = 2.3$ is the last value of T_1 for which the PFD_{avg} is less than 10⁻¹, the maximum allowable PFD_{avg} for SIL 1 in Table 1.

This conclusion would apply to this SIS at any "low" demand rate which, for Table 1, is a demand rate of 1/year or less. However, according to PAC(t), the SIS will reach the maximum allowable PAC at $T_1 = 2$ years when the demand is 1/year, but would reach the maximum allowable PAC at $T_1 = 4$

years when the demand is 1/5 years and at about $T_1 = 5.7$ years when demand is $1/10$ years. Clearly, PFD_{avg} overestimates the performance of the SIS subject to a demand of 1/year but underestimates the performance of the same SIS when demand is lower, thus leading to over-design for truly low demands.

Figure 4. Results from Example 1.

Now consider the performance of this SIS subject to "high" demand. In this case, with $\lambda_D = 10^{-4}$, the SIS does not qualify for any SIL rating using the current FOM in Table 2. For demands as high as 1/month, its PAC behavior is within the same range as the PAC(t) for an SIS with $\lambda_D = 10^{-5}$ under conditions of continuous demand, yet it is denied a SIL 1 level by the rules governing Table 2 for high demand.

Clearly PAC(t) is useful in more accurately characterizing the behavior of a given SIS under different demand conditions. Furthermore, it allows for easy evaluation of the periodic inspection interval, T_I , required before its behavior exceeds the maximum allowable PAC for a given SIL.

5.2 *Example 2*

This example examines the effects of diagnostic coverage on the SIL rating of an SIS under "high" demand. Under the assumption of continuous demand, diagnostic coverage does not affect the safety performance of a simplex system because as soon an SIS dangerous failure occurs, so does an accident. However, even under conditions of continuous demand, the safety performance of a redundant SIS can be improved by diagnostic coverage in conjunction with on-line repair. Furthermore, under conditions of high but not continuous demand, even the safety performance of a simplex system can benefit from the impact of diagnostic coverage and on-line repair. The specifications for SIL Table 2 do not provide for including the effects of diagnostic coverage with the possible exception of working it into an approximation for λ_D for a redundant system (which would not even be appropriately modeled by a constant failure rate).

In this example, λ_{p} , λ_{D} , and μ_{DD} are held fixed at the values indicated in Table 4 while C is varied between 0 and 0.9. The results are shown in Figure 5. With $\lambda_p = 1$ /week and λ_D = 5*10⁻⁶, the SIS qualifies for a rating of SIL 1 based on Table 2. Indeed, with $C = 0$ or $C = 0.5$, the PAC(t) falls

Figure 5. Results from Example 2.

Figure 6. Results from Example 3.

within the SIL 1 range for more than the first year. However, with $C = 0.9$, the PAC(t) falls within the SIL 2 range for the first year and a half. Thus, with $C = 0.9$, the safety performance of the SIS clearly exceeds the SIL that it would be assigned based on Table 2. Using PAC(t) as the FOM and determining SIL from Table 3 allows for this distinction.

5.3 *Example 3*

This example examines the effects of the on-line repair rate, μ_{DD} , on the safety performance of the SIS. The results are shown in Figure 6. With $\lambda_p = 1/m$ onth and $\lambda_D = 5*10^{-6}$, the SIS qualifies for a rating of SIL 1 according to Table 2. However, based on PAC(t) and Table 3, it clearly qualifies for a rating of SIL 2. In this particular example, changes in μ_{DD} from 1/week to 1/(2 hrs) lead to small improvements in the safety performance of the SIS, but the improvements are not significant enough to qualify the SIS for a higher SIL rating.

In this example, it would be a management decision to determine if the additional safety performance justified the costs associated with increasing the on-line repair rate. However, it should be noted that using PAC(t) as the FOM provides the information required to make such trade-offs.

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